

Важнейшие равносильные преобразования

$$1. \sqrt{f(x)} = g(x) \Leftrightarrow \begin{cases} g(x) \geq 0, \\ f(x) = g^2(x). \end{cases}$$

$$2. \sqrt{f(x)} = \sqrt{g(x)} \Leftrightarrow \begin{cases} g(x) \geq 0, \\ f(x) = g(x) \end{cases} \Leftrightarrow \begin{cases} f(x) \geq 0, \\ f(x) = g(x). \end{cases}$$

$$3. \frac{\sqrt{f(x)}}{g(x)} \geq 0 \Leftrightarrow \begin{cases} f(x) = 0, \\ g(x) \neq 0; \\ f(x) > 0, \\ g(x) > 0. \end{cases}$$

$$5. \sqrt{f(x)} < g(x) \Leftrightarrow \begin{cases} g(x) > 0, \\ f(x) \geq 0, \\ f(x) < g^2(x). \end{cases}$$

$$6. \sqrt{f(x)} \leq \sqrt{g(x)} \Leftrightarrow \begin{cases} f(x) \geq 0, \\ f(x) \leq g(x). \end{cases}$$

$$4. \sqrt{f(x)} > g(x) \Leftrightarrow \begin{cases} g(x) < 0, \\ f(x) \geq 0; \\ g(x) \geq 0, \\ f(x) > g^2(x). \end{cases}$$

$$7. |f(x)| = g(x) \Leftrightarrow \begin{cases} g(x) \geq 0, \\ f(x) = g(x), \\ f(x) = -g(x). \end{cases}$$

$$8. |f(x)| = |g(x)| \Leftrightarrow (f(x))^2 = (g(x))^2 \Leftrightarrow \begin{cases} f(x) = g(x), \\ f(x) = -g(x) \end{cases} \Leftrightarrow \begin{cases} f(x) - g(x) = 0, \\ f(x) + g(x) = 0. \end{cases}$$

$$9. |f(x)| < g(x) \Leftrightarrow -g(x) < f(x) < g(x) \Leftrightarrow \begin{cases} f(x) < g(x), \\ f(x) > -g(x). \end{cases}$$

$$10. |f(x)| > g(x) \Leftrightarrow \begin{cases} f(x) > g(x), \\ f(x) < -g(x). \end{cases}$$

$$11. |f(x)| < |g(x)| \Leftrightarrow (f(x))^2 < (g(x))^2 \Leftrightarrow (f(x) - g(x))(f(x) + g(x)) < 0.$$

$$12. a^{f(x)} > a^{g(x)} \Leftrightarrow (a-1)(f(x) - g(x)) > 0. \text{ или } a^{f(x)} \geq a^{g(x)} \Leftrightarrow (a-1)(f(x) - g(x)) \geq 0.$$

$$13. \frac{a^{f(x)} - a^{g(x)}}{h(x)} \geq 0 \Leftrightarrow \frac{(a-1)(f(x) - g(x))}{h(x)} \geq 0. \text{ или } \frac{a^{f(x)} - a^{g(x)}}{h(x)} \leq 0 \Leftrightarrow \frac{(a-1)(f(x) - g(x))}{h(x)} \leq 0.$$

$$14. (a(x))^{f(x)} = (a(x))^{g(x)} \Leftrightarrow \begin{cases} a(x) = 1, \\ a(x) > 0, \\ f(x) = g(x). \end{cases}$$

$$15. (a(x))^{f(x)} > (a(x))^{g(x)} \Leftrightarrow \begin{cases} a(x) > 0, \\ (a(x) - 1)(f(x) - g(x)) > 0. \end{cases} \text{ или}$$

$$(a(x))^{f(x)} \geq (a(x))^{g(x)} \Leftrightarrow \begin{cases} a(x) > 0, \\ (a(x) - 1)(f(x) - g(x)) \geq 0. \end{cases}$$

$$16. \log_a f(x) > 0 \Leftrightarrow \begin{cases} f(x) > 0, \\ (a-1)(f(x) - 1) > 0. \end{cases} \text{ или } \log_a f(x) < 0 \Leftrightarrow \begin{cases} f(x) > 0, \\ (a-1)(f(x) - 1) < 0. \end{cases}$$

$$17. \log_a f(x) > \log_a g(x) \Leftrightarrow \begin{cases} f(x) > 0, \\ g(x) > 0, \\ (a-1)(f(x)-g(x)) > 0. \end{cases} \quad \text{или}$$

$$\log_a f(x) < \log_a g(x) \Leftrightarrow \begin{cases} f(x) > 0, \\ g(x) > 0, \\ (a-1)(f(x)-g(x)) < 0. \end{cases}$$

$$18. \log_a f(x) = \log_a g(x) \Leftrightarrow \begin{cases} f(x) = g(x), \\ \left[\begin{array}{l} f(x) > 0, \\ g(x) > 0. \end{array} \right. \end{cases}$$

$$19. \log_{a(x)} f(x) = \log_{a(x)} g(x) \Leftrightarrow \begin{cases} f(x) = g(x), \\ a(x) > 0, \\ a(x) \neq 1, \\ \left[\begin{array}{l} f(x) > 0, \\ g(x) > 0. \end{array} \right. \end{cases}$$

$$20. \log_{a(x)} f(x) > 0 \Leftrightarrow \begin{cases} a(x) > 0, \\ f(x) > 0, \\ (a(x)-1)(f(x)-1) > 0. \end{cases} \quad \text{или} \quad \log_{a(x)} f(x) < 0 \Leftrightarrow \begin{cases} a(x) > 0, \\ f(x) > 0, \\ (a(x)-1)(f(x)-1) < 0. \end{cases}$$

$$21. \log_{a(x)} f(x) \geq 0 \Leftrightarrow \begin{cases} a(x) > 0, \\ a(x) \neq 1, \\ f(x) > 0, \\ (a(x)-1)(f(x)-1) \geq 0. \end{cases} \quad \text{или} \quad \log_{a(x)} f(x) \leq 0 \Leftrightarrow \begin{cases} a(x) > 0, \\ a(x) \neq 1, \\ f(x) > 0, \\ (a(x)-1)(f(x)-1) \leq 0. \end{cases}$$

$$22. \log_{a(x)} f(x) < \log_{a(x)} g(x) \Leftrightarrow \begin{cases} a(x) > 0, \\ f(x) > 0, \\ g(x) > 0, \\ (a(x)-1)(f(x)-g(x)) < 0. \end{cases}$$

$$23. \log_{a(x)} f(x) \geq \log_{a(x)} g(x) \Leftrightarrow \begin{cases} a(x) > 0, \\ a(x) \neq 1, \\ f(x) > 0, \\ g(x) > 0, \\ (a(x)-1)(f(x)-g(x)) \geq 0. \end{cases} \quad \text{или}$$

$$\log_{a(x)} f(x) \leq \log_{a(x)} g(x) \Leftrightarrow \begin{cases} a(x) > 0, \\ a(x) \neq 1, \\ f(x) > 0, \\ g(x) > 0, \\ (a(x)-1)(f(x)-g(x)) \leq 0. \end{cases}$$